Vertex Splitter for Straight Skeletons in 3-space

Spatial Decompositions and Graphs (VORONOI)

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Contents

Straight skeletons of 3-dimensional polyhedra

- The straight skeleton in 3-space is defined by a shrinking process. Each plane is parallelly shifted inwards at the same speed.
- The straight skeleton in the plane was definded by Aichholzer & Aurenhammer [AAAG95].
- First work on the straight skeleton for 3-dimensional polyhedra was done by Barequet et al. [BEGV08].
- Jonas Martinez implemented the skeleton computation of orthogonal polyhedra. [MVPG11]

- Before the shrinking process can continuously move all planes, each vertex needs to be split into vertices of degree 3.

Motivation

Each vertex needs to be split into vertices of degree 3, such that the surface of the shrunken polyhedron does not intersect itself.

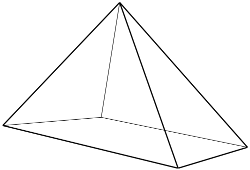

Figure: Vertex of deg. 4

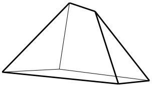

Figure: 2 vertices of deg. 3, valid surface

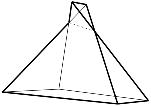

Figure: 2 vertices of deg. 3, invalid surface

Splitting convex vertices

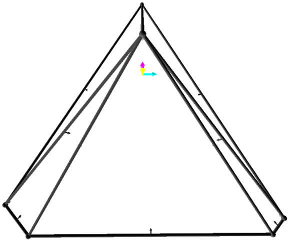

Figure: Convex vertex (deg. 5)

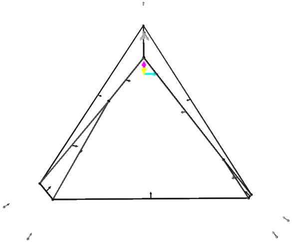

Figure: Splitted vertex

- For each adjacent face of the given vertex:
  - Calculate the offset plane of adjacent faces (3 planes)
  - Determine the distance between the intersection point of the offset planes and the vertex
- Split the vertex using the farthest distance

Combinatorial vertex splitter

Vertices of degree 3 span an unrooted binary tree.

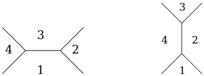

Figure: Combinations for deg. 4

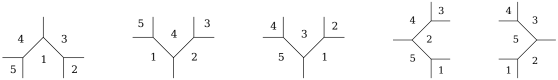

Figure: Combinations for deg. 5

Number of possible unrooted binary trees

$$C_n := \frac{1}{n+1}\binom{2n}{n} = O(c^n), c > 1$$

Check for self-intersections

- All facets need to be self-intersection free.
- No edges are allowed to intersect any other facet.

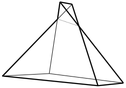

Figure: Invalid surface: self-intersecting facets

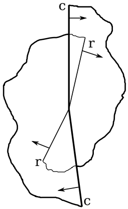

Figure: 2 tilted facets

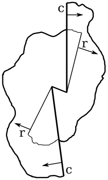

Figure: Valid solution

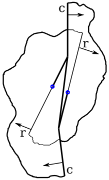

Figure: Self-intersecting polyhedron

Saddle points

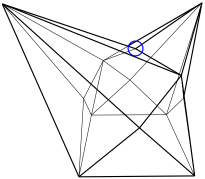

Figure: Front view

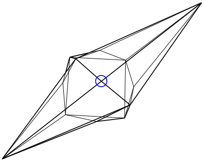

Figure: Top view

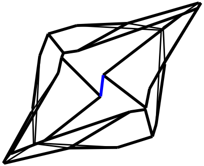

Figure: Convex solution

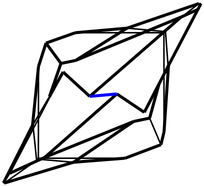

Figure: Reflex solution

- This example proves that offsetting a polyhedron is not unique.

Events

There are 12 events

- Vanish events (like 2-dim. edge event)
  - 1 up to 6 edges vanish simultaneously (even if the planes are tilted)
- Contact events (like 2-dim. split event)
  - Vertex-Vertex contact
  - Vertex-Edge contact (2 events)
  - Edge-Edge contact (2 events, non-local)
  - Vertex-Facet contact (non-local)

For convex polyhedra sufficient

- Vanish events
  - Edge event (1 edge)
  - Triangle event (3 edges)
  - Tetrahedron event (6 edges)

Central projection onto the sphere

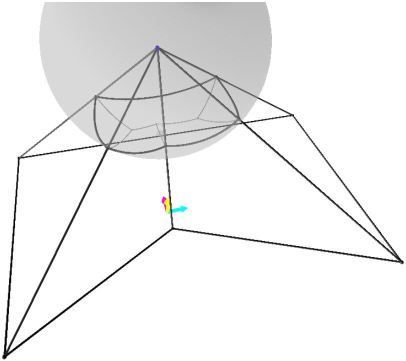

Figure: Central projection onto the sphere

Complexity bounds

Description	Bound	Reference
Straight skel. of convex polyhedra	$\Omega(n^2)$	[Hel94]
Straight skel. of convex polyhedra	$O(n^2)$	[BEGV08]
Straight skel. of general polyhedra	$\Omega(n^2\alpha^2(n))$	[BEGV08]
Straight skel. of general polyhedra	$O(n^4)$	
estim. time to find next event	$O(nr^2)$	

Implementation

- Implemented in C++ (approx. 35k LOC)
- Our geometry kernel has double precision only
  CGAL's kernel can optionally be linked
- Resulting skeleton is stored using a relational database (SQLite)
- Interactive animation is done using OpenGL
- Software rendering for PostScript output
- Automated testing

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Conclusion & Outlook

Conclusion

- The offset of a polyhedron is not unique.
- We've found 12 events.
- The vertex splitter shows how to handle these events.
- Our work solved open problems. [BEGV08]

Outlook

- We are working on a more "elegant" way to split vertices.

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