Remoteness, proximity and few other distance invariants in graphs

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4. Suggest ideas of proof.

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[4]M. Aouchiche, P. Hansen, Proximity and remoteness in graphs: results and conjectures, Networks 58 (2) (2011) 95–102.

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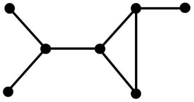

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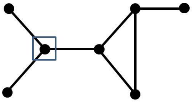

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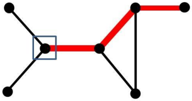

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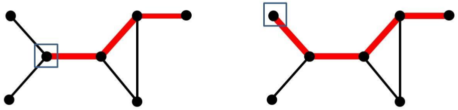

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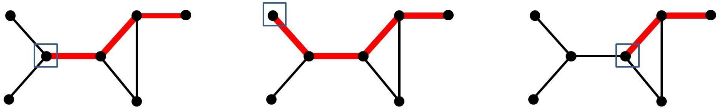

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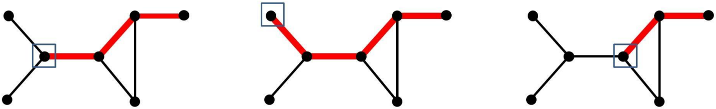

The *radius* r, *diameter* D and the *average eccentricity* ecc of G are now defined as

$$r(G) = \min_{v \in V} e(v), \;\; D(G) = \max_{v \in V} e(v), \;\; ecc(G) = \frac{1}{n} \sum_{v \in V} e(v).$$

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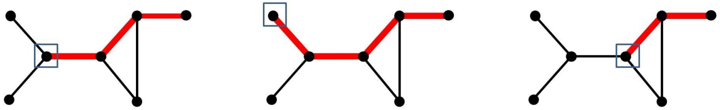

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The *center* of a graph is a vertex v of minimum eccentricity.

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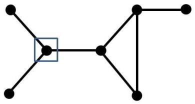

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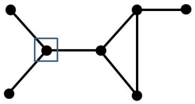

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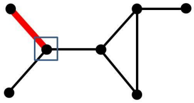

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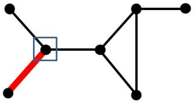

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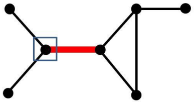

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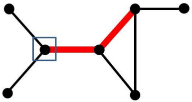

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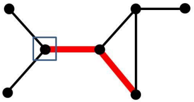

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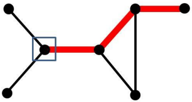

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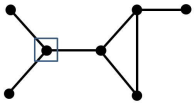

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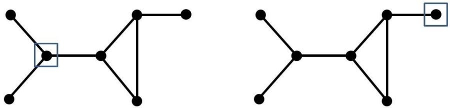

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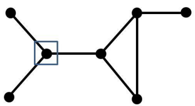

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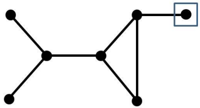

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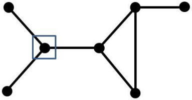

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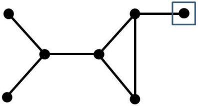

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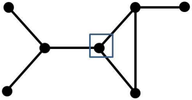

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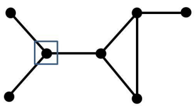

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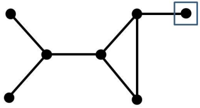

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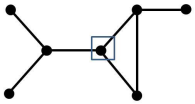

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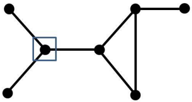

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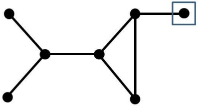

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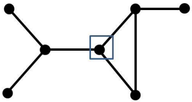

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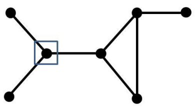

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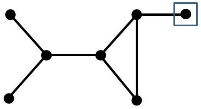

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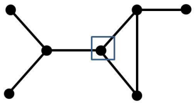

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A vertex $v \in V$ is *centroidal* if $\pi(v) = \pi(G)$

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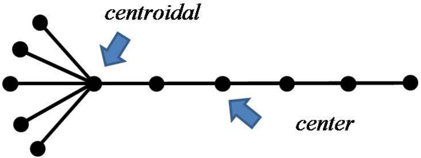
Generally: center $\neq$ centroidal.



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Conjecture 2. Let G be a connected graph on $n \geq 3$. Then

$$ecc - \rho \leq \begin{cases} \frac{3n+1}{4}\frac{n-1}{n} - \frac{n}{2} & \text{if } n \text{ is odd,} \\ \frac{n-1}{4} - \frac{1}{4n-4} & \text{if } n \text{ is even,} \end{cases}$$

with equality if and only if G is a cycle C_n.

Conjecture 3. Let G be a connected graph on $n \geq 3$ vertices. Then

$$\rho - r \geq \begin{cases} \frac{3-n}{4} & \text{if } n \text{ is odd,} \\ \frac{n^2}{4n-4} - \frac{n}{2} & \text{if } n \text{ is even.} \end{cases}$$

The inequality is best possible as shown by the cycle C_n if n is even and by the graph composed by the cycle C_n together with two crossed edges on four successive vertices of the cycle.

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- the proof that Conjectures 2 and 3 hold for trees.

Main results

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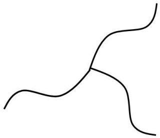

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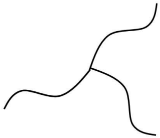

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$$\bar{l}(G) - \pi(G) = \frac{7n^2+13n-2}{27n} - \frac{2+n}{6}$$

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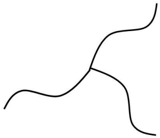

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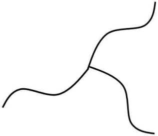
Conjecture 1. The difference $\bar{l} - \pi$ is maximum for a graph G composed of three paths of almost equal lengths with a common end point.

Lema 1. The difference $\bar{l} - \pi$ is greater for a tree G on n vertices consisting of three paths of almost equal length with a common end point than for path P_n.



For $n = 3k + 1$:

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For n even:

$$\bar{l}(P_n) - \pi(P_n) = \frac{1+n}{3} - \frac{n^2}{4(n-1)}$$

Main results

Lema 2. The difference $\bar{l} - \pi$ is greater for a tree G on n vertices consisting of three paths of almost equal length with a common end point than for a tree G' on n vertices consisting of four paths of equal length.

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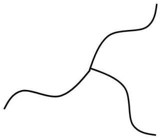

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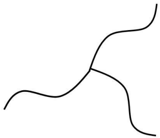

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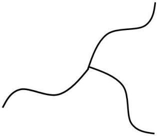

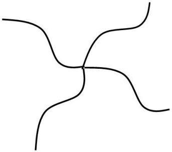

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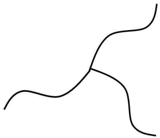

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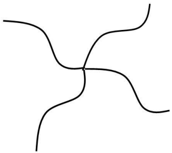

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$$\bar{l}(G') - \pi(G') = \frac{5n^2+14n-3}{24n} - \frac{n+3}{8}$$

Main results

Lema 3. Let G be a tree on $n \geq 6$ vertices with at least four leafs. Then there is a tree G' on n vertices with three leafs for which the difference $\bar{l} - \pi$ is greater or equal than for G.

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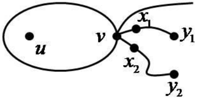

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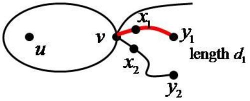

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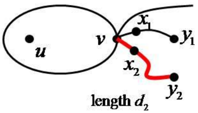

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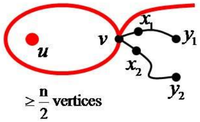

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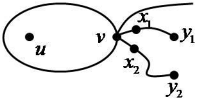

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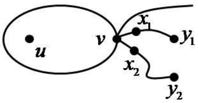

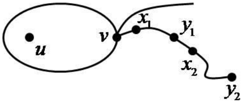

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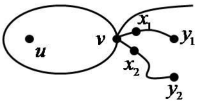

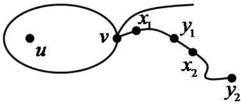

From here we obtain

$$\bar{l}(G') - \pi(G') \geq \bar{l}(G) - \pi(G) + \frac{d_1 d_2}{n-1} \left(\frac{2(n - d_1 - d_2 - 1)}{n} - 1 \right).$$

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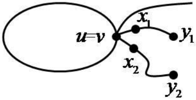

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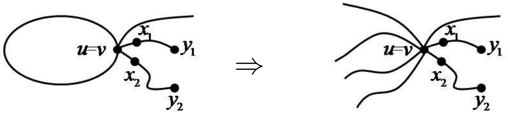

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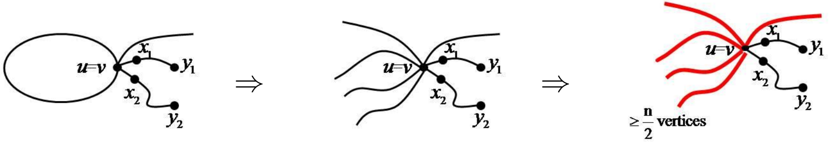

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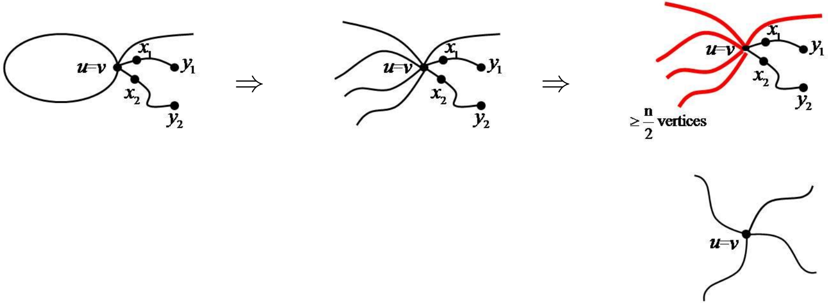

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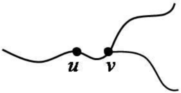

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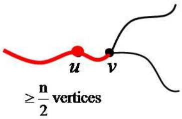

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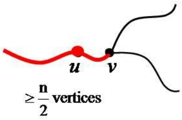

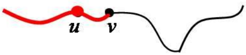

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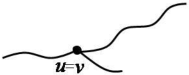

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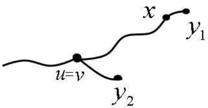

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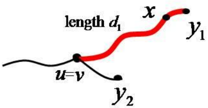

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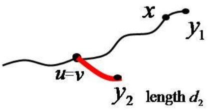

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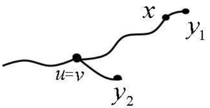

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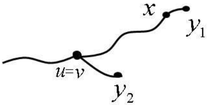

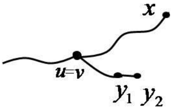

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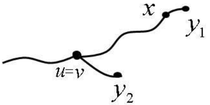

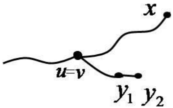

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Theorem 5. Among all trees on $n \geq 4$ $(n \neq 5)$ vertices with average distance $\bar{l}$ and proximity π, the difference $\bar{l} - \pi$ is maximal for a tree G composed of three paths of almost equal lengths with a common end vertex.

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$$\begin{aligned} \pi(G) &= \pi(u) = \pi'(u) \geq \pi(G') \\ \bar{l}(G) &\leq \bar{l}(G') \end{aligned}$$

Main results

Conjecture 2. Let G be a connected graph on $n \geq 3$. Then

$$ecc - \rho \leq \begin{cases} \frac{3n+1}{4}\frac{n-1}{n} - \frac{n}{2} & \text{if } n \text{ is odd,} \\ \frac{n-1}{4} - \frac{1}{4n-4} & \text{if } n \text{ is even,} \end{cases}$$

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Theorem. Among trees on $n \geq 3$ vertices, the difference $ecc - \rho$ is maximal for path P_n.

Main results

Conjecture 2. Let G be a connected graph on $n \geq 3$. Then

$$ecc - \rho \leq \begin{cases} \frac{3n+1}{4}\frac{n-1}{n} - \frac{n}{2} & \text{if } n \text{ is odd,} \\ \frac{n-1}{4} - \frac{1}{4n-4} & \text{if } n \text{ is even,} \end{cases}$$

with equality if and only if G is a cycle C_n.

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Now, from

$$ecc(P_n) - \rho(P_n) = \begin{cases} \frac{n-2}{4} & \text{for even } n, \\ \frac{n}{4} - \frac{2n+1}{4n} & \text{for odd } n. \end{cases}$$

easily follows that Conjecture 2 holds for trees.

Main results

Conjecture 3. Let G be a connected graph on $n \geq 3$ vertices. Then

$$\rho - r \geq \begin{cases} \frac{3-n}{4} & \text{if } n \text{ is odd,} \\ \frac{n^2}{4n-4} - \frac{n}{2} & \text{if } n \text{ is even.} \end{cases}$$

The inequality is best possible as shown by the cycle C_n if n is even and by the graph composed by the cycle C_n together with two crossed edges on four successive vertices of the cycle.

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Theorem. Let G be a tree on n vertices. If n is odd, then the difference $\rho - r$ is less or equal for path P_n then for G. If n is even, then the difference $\rho - r$ is less or equal for path P_{n-1} with a leaf appended to a central vertex than for G.

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For a path P_n (n odd) it holds that $\rho - r = \frac{1}{2}$.
For a path P_{n-1} (n even) with a leaf appended to a central vertex G it holds that $\rho - r = \frac{n}{2(n-1)}$.

Main results

Thank you for your attention.