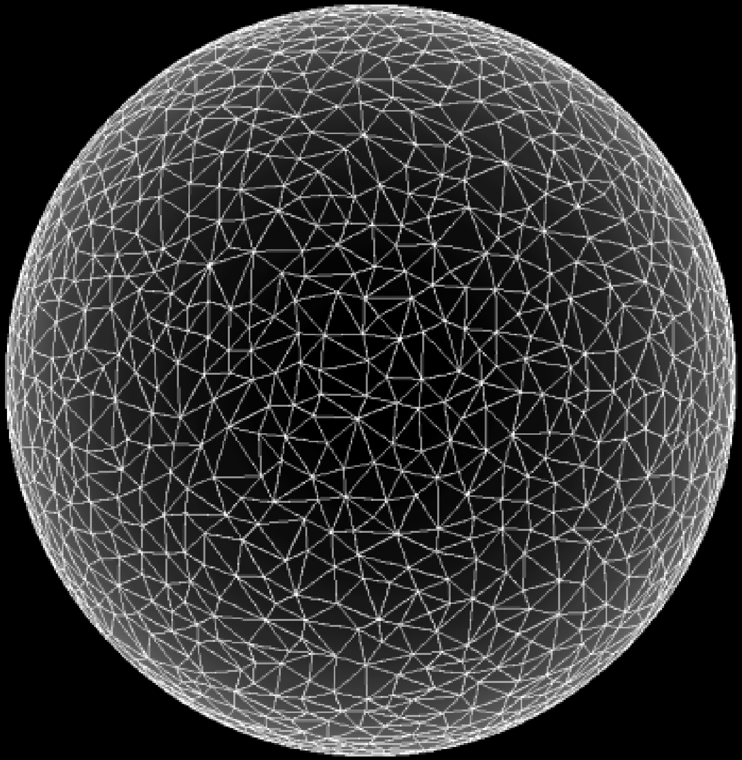
Graph coloring with geometric flavor



Jarek Grytczuk

Jagiellonian University, Kraków
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Nonrepetitive coloring of graphs

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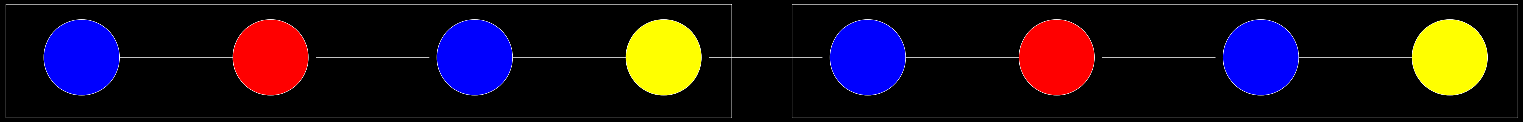

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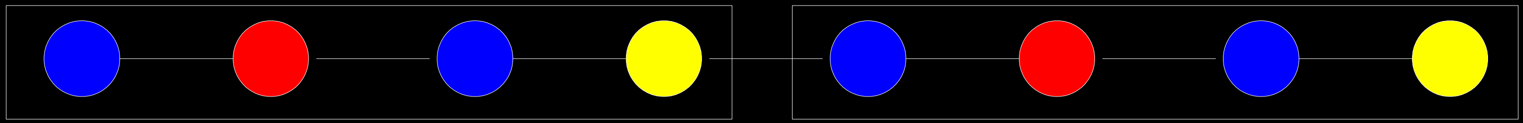

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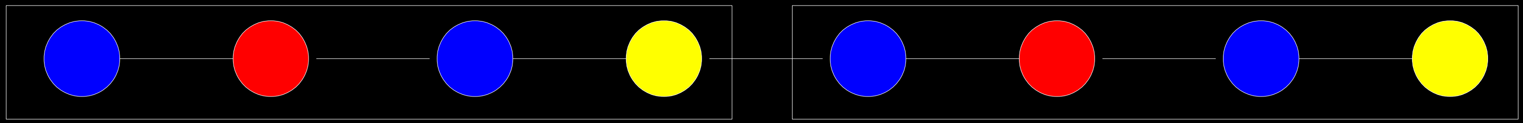

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Notation: $\pi(G)$ = the least number of colors in a nonrepetitive coloring of G.

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Dujmović, Joret, Kozik, Wood (2013): Graphs of pathwidth at most k satisfy

$$\pi(G) \leq 2k^2 + 6k + 1.$$

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Conjecture: Every planar graph has a 4-coloring with no 10^{49}-repetitive paths.

Lines on the plane

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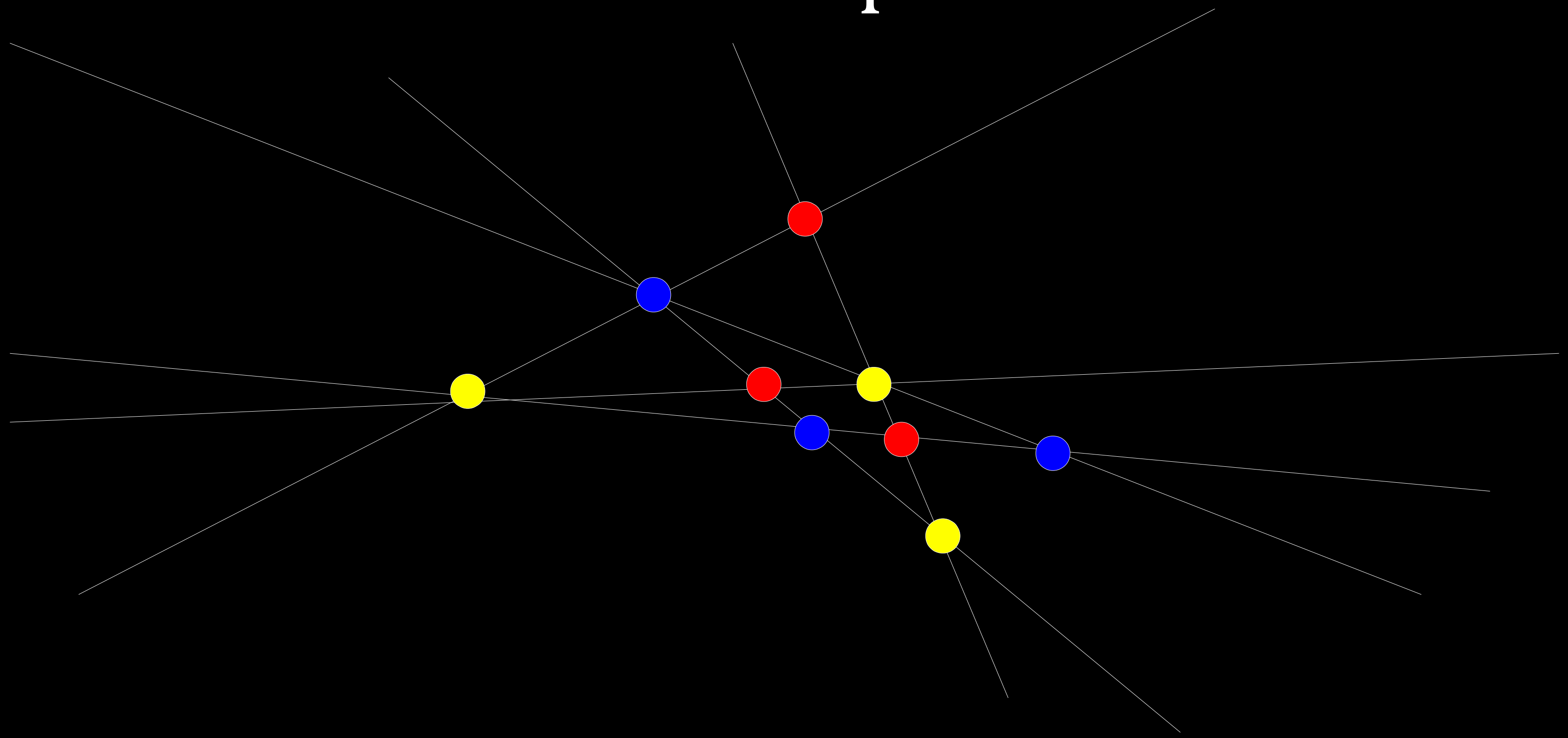

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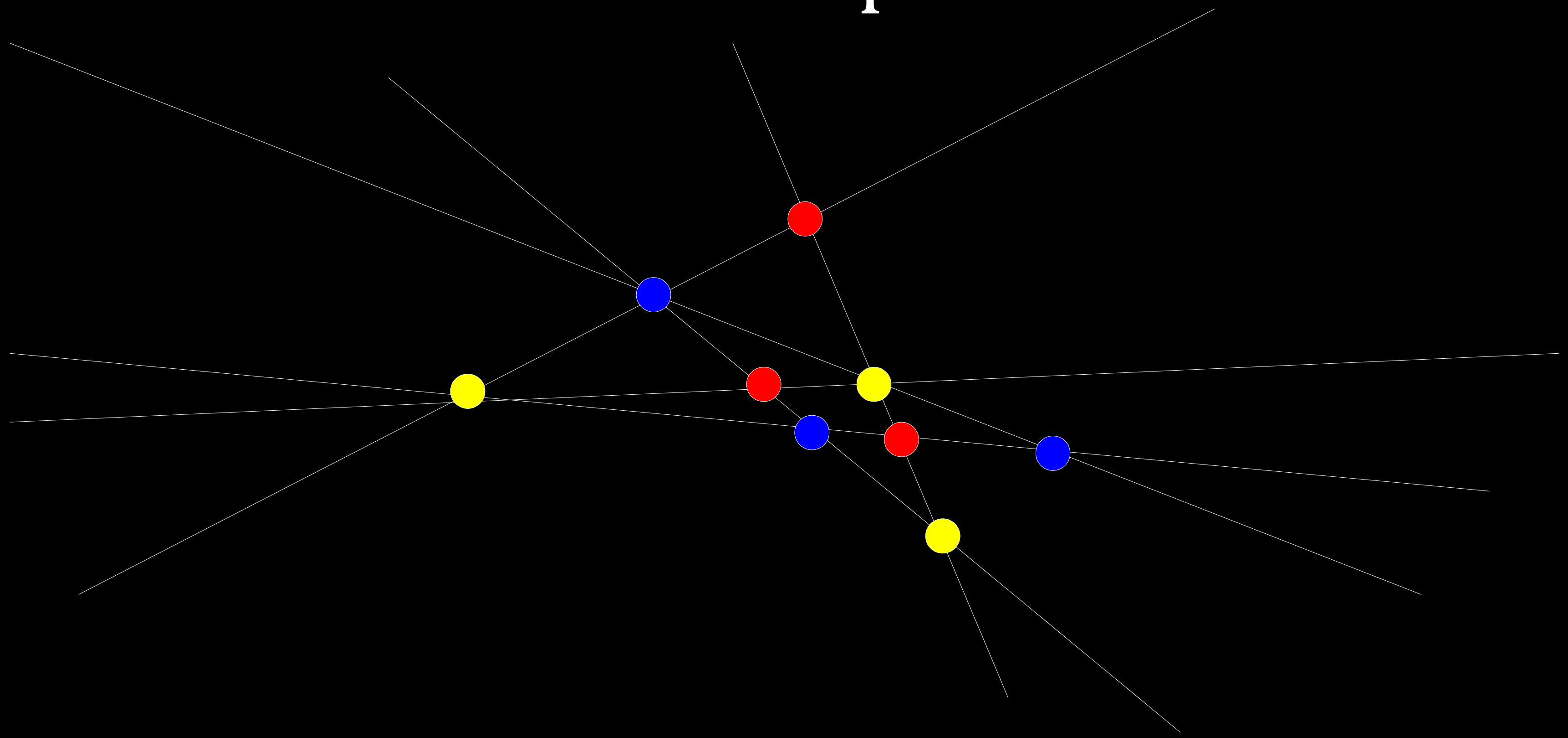

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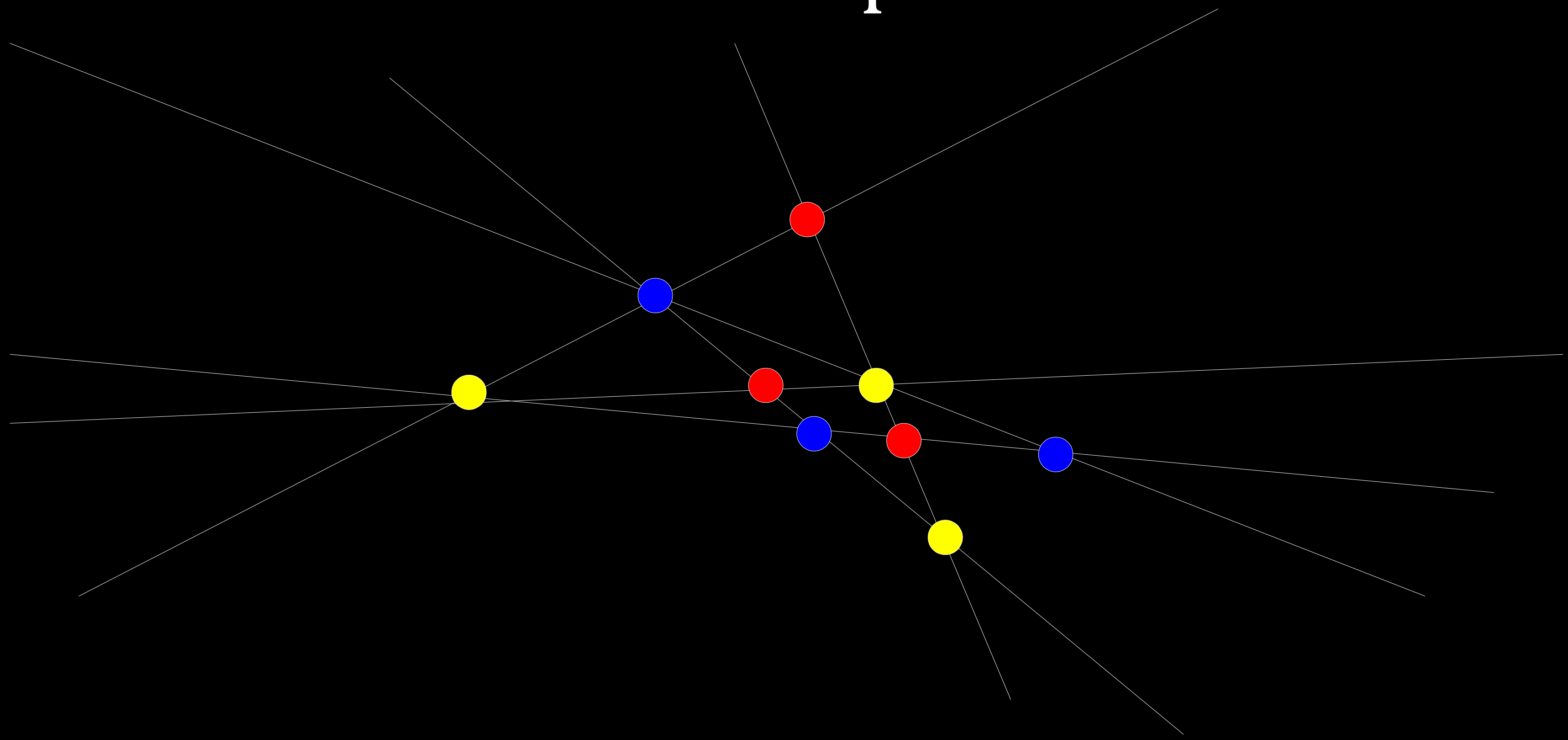

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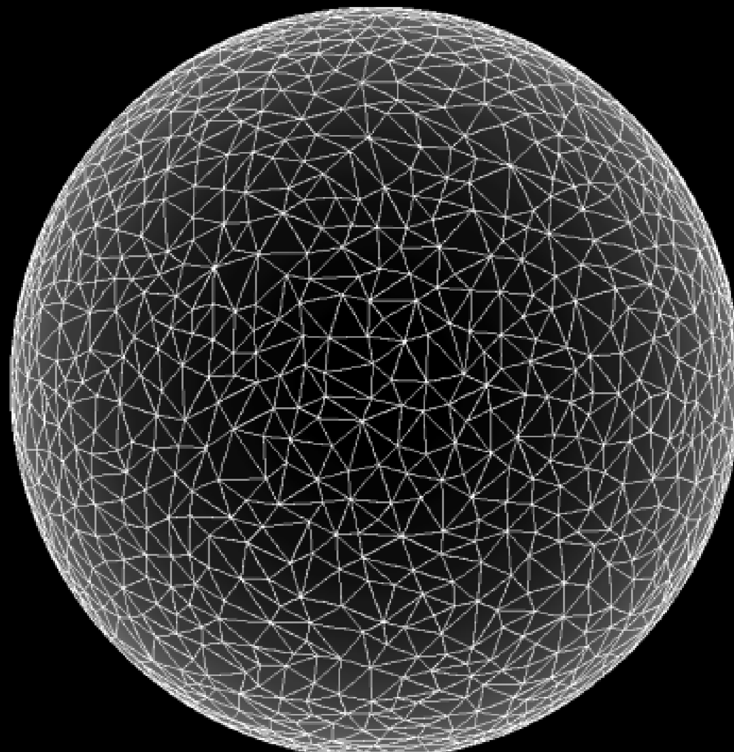
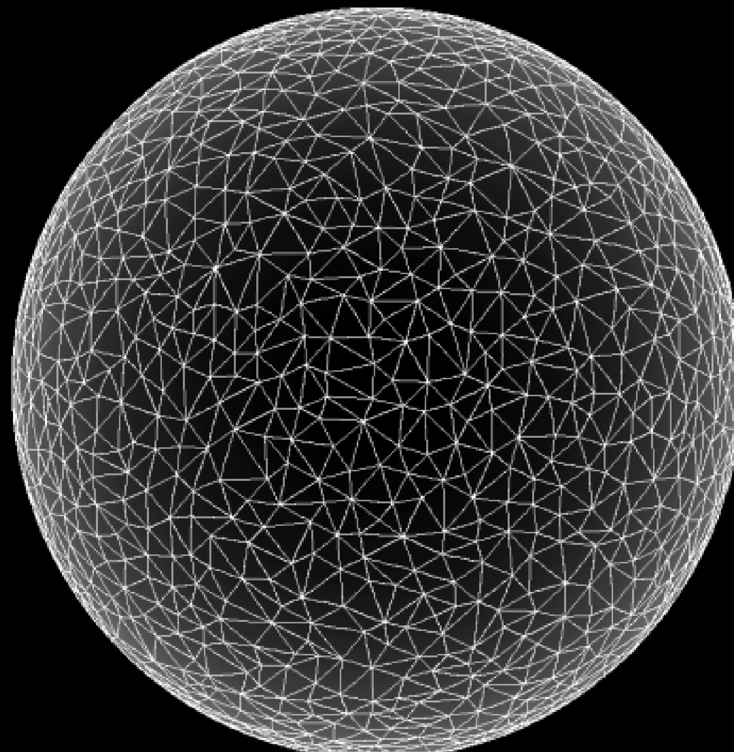
Grytczuk, Zmarz (2014): Every line arrengement has a nonrepetitive coloring using 405 colors.





Thank You!