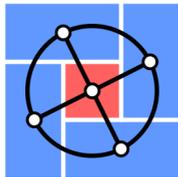


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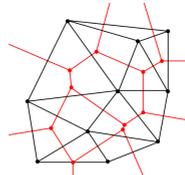
# EuroGIGA Final Conference

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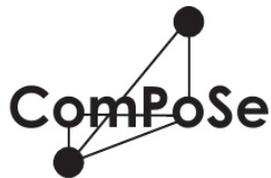
Freie Universität Berlin  
February 17-21, 2014



**GraDR**



**VORONOI**



BOOKLET OF ABSTRACTS

**E**UROPEAN  
**S**CIENCE  
**F**OUNDATION

Freie Universität  Berlin



## **Program**

### **Monday, February 17, 2014**

12:00–14:30 Registration  
13:00–14:30 Lunch  
14:30–16:00 Talks  
16:00–16:30 Coffee break  
16:30–18:00 Talks

### **Tuesday, February 18, 2014**

09:30–11:00 Talks  
11:00–11:30 Coffee break  
11:30–12:30 Invited talk – Raimund Seidel (Saarbrücken)  
12:30–14:00 Lunch break  
14:00–15:30 Talks  
15:30–16:00 Coffee break  
16:00–17:30 Talks

### **Wednesday, February 19, 2014**

09:30–11:00 Talks  
11:00–11:30 Coffee break  
11:30–12:30 Invited talk – Natan Rubin (Paris)  
12:30–14:00 Lunch break  
14:00–15:30 Talks  
16:00–23:00 Excursion and Conference dinner

### **Thursday, February 20, 2014**

09:30–11:00 Talks  
11:00–11:30 Coffee break  
11:30–12:30 Invited talk – Wilfried Imrich (Leoben)  
12:30–14:00 Lunch break  
14:00–15:30 Talks  
15:30–16:00 Coffee break  
16:00–18:00 Talks  
18:00–19:00 Business meeting

### **Friday, February 21, 2014**

09:30–11:00 Talks  
11:00–11:30 Coffee break  
11:30–12:30 Invited talk – Giuseppe Liotta (Perugia)  
12:30–14:00 Lunch and goodbye



## Abstracts

### Invited talks

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**Tuesday, 11:30 - 12:30** (Chair: Oswin Aichholzer)

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**Raimund Seidel** (Universität des Saarlandes)

*Counting and Estimating Planar Structures*

How many straight edge triangulations does a given set  $S$  of  $n$  points in the plane admit? How many plane straight edge spanning trees? How many non-crossing straight edge matchings? How many plane straight edge Hamiltonian cycles? Questions of counting, bounding, or estimating the number of such planar structures or enumerating them have been considered for quite a while, but alas with only limited success. I will discuss some recent progress in this area, which has resulted in counting methods that are provably faster than enumeration.

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**Wednesday, 11:30 - 12:30** (Chair: Günter Rote)

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**Natan Rubin** (Université Pierre et Marie Curie Paris)

*On kinetic Delaunay triangulations*

Given a finite set  $P$  of points in the plane, three points of  $P$  form a Delaunay triangle if their circumscribing circle contains no further points of  $P$ . These triangles form the famous Delaunay triangulation which, along with its dual Voronoi diagram, is central to Combinatorial and Computational Geometry.

Despite several decades of intensive study, our understanding of the combinatorial behaviour of Voronoi and Delaunay structures is still far from satisfactory. For example, if the points of  $P$  are moving along lines at uniform speed, it has been long conjectured that their Delaunay triangulation experiences at most near-quadratically many discrete changes during the motion.

Our recent study affirms the above conjecture. This is done in a far more general, purely topological setting. We discuss this work in connection with some other fundamental results on the combinatorial complexity of geometric structures.

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**Thursday, 11:30 - 12:30** (Chair: Tomáš Pisanski)

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**Wilfried Imrich** (Montanuniversität Leoben)

*Symmetry Breaking*

A coloring of the vertices of a graph  $G$  is called distinguishing if the trivial automorphism is the only automorphism of  $G$  that preserves the coloring. There is a vast literature on distinguishing finite graphs. Often it is important to have a lower bound on the number of vertices that is moved by every nontrivial automorphism.

For infinite graphs Tom Tucker conjectured that, if every automorphism of a connected, locally finite graph moves infinitely many vertices, then there exists a distinguishing coloring that uses only two colors. This is known as the Infinite Motion Conjecture. Despite many intriguing partial results, it is still open in general.

This conjecture, its generalizations to uncountable graphs, to groups acting on structures, and to endomorphisms of countable and uncountable graphs, has become a widely studied topic.

In this talk I will present a short overview on how to distinguishing finite graphs, and an account of the Infinite Motion Conjecture and several of its variants. I will also describe some of the methods used, and include results on minimizing one of the color classes in distinguishing colorings with 2 colors.

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**Friday, 11:30 - 12:30** (Chair: Jan Kratochvíl)

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**Giuseppe Liotta** (Università degli Studi di Perugia)

*Graph Drawing Beyond Planarity: Some results and open problems*

Recent technological advances have generated torrents of relational data sets that are often represented and visually analyzed as graphs drawn in the plane. The large size of these data sets poses fascinating challenges to graph drawers both from a practical and from a theoretical point of view: while a considerable portion of the existing graph drawing literature showcases elegant algorithms and sophisticated data structures under the assumption that the input graph is planar, most graphs are in fact non-planar in practice. In this talk I will briefly review recent findings and outline some emerging research directions about the theory of “nearly planar” graphs, i.e. graphs that have drawings where some crossing configurations are forbidden.

## Project talks

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Monday, 14:30 - 16:00 (Chair: Ferran Hurtado)

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**Birgit Vogtenhuber** (Graz University of Technology)

*Empty triangles in good drawings of the complete graph*

A good drawing of a simple graph is a drawing on the sphere or, equivalently, in the plane in which vertices are drawn as distinct points, edges are drawn as Jordan arcs connecting their end vertices, and any pair of edges intersects at most once. In any good drawing, the edges of three pairwise connected vertices form a Jordan curve which we call a triangle. We say that a triangle is empty if one of the two connected components it induces does not contain any of the remaining vertices of the drawing of the graph. We show that the number of empty triangles in any good drawing of the complete graph  $K_n$  with  $n$  vertices is at least  $n$ .

**Stefan Felsner** (Technische Universität Berlin)

*Intersection Graphs and Order Dimension*

It is well known that there is a rich interplay between containment graphs and order dimension. In this talk I discuss some recent fruitful connections between intersection graphs and order dimension.

**Javier Tejel** (University of Zaragoza)

*On 4-connected geometric graphs*

A *geometric graph* is a graph whose vertices are distinct points in the plane and whose edges are straight line segments between pairs of vertices. A *plane graph* is a geometric graph in which no two edges cross. A set of points is *k-connectible* if it admits a *k-connected plane graph* on it. For  $k = 1, 2, 3$ , it is well-known when a set of points is *k-connectible* and how to build a *k-connected plane graph* on it. For  $k = 4$ , Dey, Dillencourt, Ghosh and Cahill gave an  $O(n \log n)$  algorithm to build a 4-connected triangulation on every set of points whose convex hull is a triangle, provided that a certain condition is satisfied.

In this talk, we study the problem of recognizing whether a set of points is 4-connectible. We show a new condition that must be satisfied for any set of points to be 4-connectible, and a linear algorithm to test whether a set of points satisfies this condition. We also give a quadratic

algorithm to build a 4-connected triangulation on every set of points satisfying this new condition. The algorithm is based on a special partition of the plane into convex regions.

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**Monday, 16:30 - 18:00** (Chair: Franz Aurenhammer)

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**Monika Cerinšek** (Hruška d.o.o. Ljubljana)

*Social network analysis of Zentralblatt MATH data*

Our goal was to analyze the structure and development of mathematics using the social network analysis. We obtained the bibliographic data about mathematical publications in the years 1990 - 2010 from the Zentralblatt MATH database. The data were converted into a collection of 2-mode networks: works x authors, works x journals, works x keywords, works x classifications, and a partition of works by their publication year.

We developed a methodology for analyzing such collections of networks. Using network multiplication we can compute different derived networks revealing interesting aspects of the data. We also focus a part of the analysis to a specific mathematical discipline – the graph theory. We present results about our ranking of graph theorists, collaboration among them, major journals and characteristic keywords for the graph theory.

The networks were analyzed using Pajek, a computer program for analysis and visualization of large networks.

This is joint work with Vladimir Batagelj and Petra Jakovac.

**Tanja Vojkovic** (University of Split)

*Sleeper agents and distributed keys*

Organization X wants to establish a network of sleepers (spies activated on command) in the country Y. Also, they want to give them a secret message that has to be recovered on the command and should be secret to the country Y. The message is protected by the sequence of keys and all the keys have to be known in order to read the message. We can represent that situation as a graph where people are connected by an edge if they know each other. Every vertex has some of the keys to the message and the message can be reconstructed if there is a connected component of the graph that has all the keys. As a countermeasure country Y tries to infiltrate its own agents in this network of spies.

Country Y wants either to get all the keys which would enable them to read the message, or to prevent the sleepers from reading the message. Moreover, some of the sleepers may give up or be discovered by the country Y before executing their mission, we call that sleepers missing persons. Our goal is to determine if for a given number of country Y's agents and missing persons there exists a network which is resilient to their attack. We analyze the minimal number of people in that network and minimal number of keys needed. Our results include the cases of 1 agent and any number of missing persons, 2 agents and 1, 2 or 3 missing persons, and 3 agents and 1 missing person. This is joint work with Damir Vukicevic.

**Damir Vukicevic** (University of Split)

*Distributed key and agents*

In this presentation two problems will be addressed.

First, security of the messages protected by several keys which are distributed to different persons is analyzed. The system of  $n$  keys  $K = \{1, \dots, n\}$  and  $p$  persons each possessing some subset  $K_i \subseteq K$  of keys is assumed to be  $r$ -secure if there is no group of  $r$  renegade individuals that can either read the message (i.e. collect all different keys) or disable the rest of persons to read the message (i.e. collect all the copies of at least one key). We analyze for which values of  $n, p$ , and  $r$  it is possible to construct an  $r$ -secure network with  $p$  persons and  $n$  keys.

The second analyzed problem is the following: Suppose that some organization wants to plant a group of "sleepers" (spies that live normal life until they are called to perform some mission). After they get a message they have to go to a certain location and activate a secret weapon or do some diversion. Location and mission is protected by a secret code that can be unlocked by a series of keys. Each sleeper can have some (or none) of the keys. Further, each of the sleepers knows only some of his colleagues. These sleepers can be represented as a complex network in which edges connect pairs of sleepers that know each other. The mission can obviously be implemented if there is a connected component of this network that has all the keys. The aim is to make a network that is resilient to adversary agents planted in the sleepers group. It is assumed that if there is an adversary agent, he will betray all the sleepers that he knows and give his keys to the adversary. A network is  $r$ -resilient if no  $r$  persons acting as adversary agents can

neither collect all the keys nor break the graph in such components that no component has all the keys. Our aim is to determine for the given triplet of numbers  $(a, p, k)$  whether there is a network with  $p$  persons and distribution of  $k$  keys in that network such that it is resilient to the attack of any  $a$  agents.

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**Tuesday, 09:30 - 11:00** (Chair: Vera Sacristán)

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**Vincent Kusters** (ETH Zurich)

*Planar Packing of Binary Trees*

In the graph packing problem we are given several graphs and have to map them into a single host graph such that each edge of the host graph is used at most once. Much research has been devoted to the packing of trees, especially to the case where the host graph is planar. More formally, this problem can be defined as follows: Given any two trees  $T_1$  and  $T_2$  on  $n$  vertices, we want a simple planar graph  $G$  on  $n$  vertices such that the edges of  $G$  can be colored with two colors and the subgraph induced by the edges colored  $i$  is isomorphic to  $T_i$ , for  $i = 1, 2$ .

A clear exception that must be made is the star tree which cannot be packed together with any other tree. But a popular hypothesis states that this is the only exception, and all other pairs of trees admit a planar packing. Previous proof attempts lead to very limited results only, which include a tree and a spider tree, a tree and a caterpillar, two trees of diameter four and two isomorphic trees.

We make a step forward and prove the hypothesis for any two binary trees. The proof is algorithmic and yields a linear time algorithm to compute a plane packing, that is, a suitable two-edge-colored host graph along with a planar embedding for it. In addition we can also guarantee several nice geometric properties for the embedding: vertices are embedded equidistantly on the  $x$ -axis and edges are embedded as semi-circles.

**Dömötör Pálvölgyi** (ELTE Budapest)

*Indecomposable coverings with unit discs*

Let  $\mathcal{C} = \{C_i \mid i \in I\}$  be a collection of sets in  $\mathbb{R}^2$ . We say that  $\mathcal{C}$  is an  $m$ -fold covering if every point of  $\mathbb{R}^2$  is contained in at least  $m$  members of  $\mathcal{C}$ . A 1-fold covering is simply called a covering.

A planar set  $C$  is said to be *cover-decomposable* if there exists a (minimal) constant  $m = m(C)$  such that every  $m$ -fold covering of the plane with translates of  $C$  can be decomposed into two coverings.

The problem of characterizing all cover-decomposable sets in the plane was proposed by Pach in 1980. He conjectured that every planar convex set  $C$  is cover-decomposable.

We disprove this conjecture by showing it holds for no convex set with a smooth boundary, so for example the unit disc.

**Maurizio Patrignani** (Roma Tre University)

*On the Complexity of some Simultaneous and Clustered Planarity Problems*

Planarity is one of the most fascinating topics of graph theory. Two strains of planarity, both emerged in the last decades, are attracting considerable research effort: simultaneous planarity and clustered planarity. In simultaneous planarity several graphs on the same set of vertices are provided and the goal is to determine whether they admit planar drawings where the vertices have the same coordinates. It is also required that edges common to different graphs are represented with the same curve (Simultaneous Embedding with Fixed Edges), which is granted if the drawings are required to be straight-line (Geometric Simultaneous Embedding). Clustered planarity, instead, deals with clustered graphs, i.e. graphs provided with a hierarchy of clusters (recursively nested sets of vertices). It is asked to determine whether the clustered graph admits a planar drawing where each cluster is represented by a simple closed curve that encircles all and only the vertices of the cluster, that does not intersect other clusters, and that intersects each edge at most once. Despite the considerable effort of many researchers over the years, simultaneous and clustered planarity are still open in the general case. However, on one side more and more polynomial cases are discovered, on the other side, these two problems turned out to be strictly related one to the other. This talk provides a state of the art and some recent results.

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**Tuesday, 14:00 - 15:30** (Chair: Stefan Felsner)

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**Inmaculada Ventura** (Universidad de Sevilla)

*New results on the coarseness of bicolored point sets*

Let  $S$  be a set of  $n$  points in the plane. A subset  $I$  of  $S$  is an island if there exists a convex set  $C$  such that  $I = C \cap S$ . Given a 2-coloring of  $S$  (the points are colored red or blue), the *discrepancy* of an island is the absolute value of the number of red minus the number of blue points it contains. A *convex partition* of  $S$  is a partition of  $S$  into islands with pairwise disjoint convex hulls. The discrepancy of a convex partition is the discrepancy of its island of minimum discrepancy. The *coarseness* of  $S$  is the discrepancy of the convex partition of  $S$  with maximum discrepancy. This concept was recently defined by Bereg et al. [1] for measuring how blended the elements of  $S$  are. In this talk we discuss two problems related to the coarseness parameter. Namely, the problems are:

Given a 2-coloring of  $S$ , is there any polynomial-time constant approximation algorithm for computing the coarseness of  $S$ ?

Given a set of points  $S$ , how to color each point in  $S$  such that the resulting 2-colored point set has small coarseness?

For the first problem, we show that there exists a polynomial-time approximation algorithm whose ratio is between  $1/128$  and  $1/64$ . For the second problem, we prove that every  $n$ -point set  $S$  can be colored such that its coarseness is  $O(n^{1/4}\sqrt{\log n})$ . This bound is almost tight since there exist  $n$ -point sets such that every 2-coloring gives coarseness at least  $\Omega(n^{1/4})$ . Both questions were posted as open problems by Bereg et al. [1].

This is joint work with J.M. Díaz-Báñez, R. Fabila-Monroy, and P. Pérez-Lantero.

- [1] S. Bereg, J.M. Díaz-Báñez, D. Lara, P. Pérez-Lantero, C. Seara, J. Urrutia, *On the Coarseness of a Bichromatic Point Set*. Computational Geometry: Theory and Applications, Volume 46, Issue 1, 2013, Pages 65-7, 2012.

**Jean Cardinal** (Université Libre de Bruxelles)

*Making Octants Colorful and Related Covering Decomposition Problems*

We give new positive results on the long-standing open problem of geometric covering decomposition for homothetic polygons. In particular, we prove that for any positive integer  $k$ , every finite set of points in  $\mathbb{R}^3$  can be colored with  $k$  colors so that every translate of the negative octant containing at least  $k^6$  points contains at least one of each color. The best previously known bound was doubly exponential in  $k$ . This yields, among other corollaries, the first polynomial bound for the decomposability of multiple coverings by homothetic triangles. We also investigate related decomposition problems involving intervals appearing on a line. We prove that no algorithm can dynamically maintain a decomposition of a multiple covering by intervals under insertion of new intervals, even in a *semi-online* model, in which some coloring decisions can be delayed. This implies that a wide range of sweeping plane algorithms cannot guarantee any bound even for special cases of the octant problem.

**Mercè Mora** (Universitat Politècnica de Catalunya, Barcelona)

*Location and domination in graphs*

Many problems related to location and domination in graphs imply the study of special subsets of vertices and some parameters. The main goal is to determine the existence or the location of some facility, object or item by placing the minimum number of detection devices or watchers in some vertices of the graph. Different parameters are defined by considering restrictions on the detection devices. Definitions and basic properties of some of those parameters, including bounds and values on some families, will be given in this talk. I will also give some recent results about location-domination in graphs, a concept introduced by P. Slater in 1988. A subset  $S$  of vertices of a graph is a *locating-dominating set* if  $S$  is a dominating set such that every pair of vertices not in  $S$  have different neighborhoods in  $S$ . The *location-domination number* of a graph is the minimum size of a locating-dominating set. We have studied the relation of locating-dominating sets in a graph and its complement, obtaining Nordhaus-Gaddum type bounds and some results in block-cactus graphs (a family of graphs containing cycles, paths and complete graphs) and bipartite graphs. Joint work with C. Hernando and I. M. Pelayo.

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**Tuesday, 16:00 - 17:30** (Chair: Stefan Langerman)

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**Rafel Jaume** (Freie Universität Berlin)

*The least-squares partial-matching Voronoi diagram*

Given two point sets  $A, B \subset \mathbb{R}^d$  with  $k = |B| \leq |A| = n$ , we study the partial-matching problem of translating  $B$  to a position where it is best resembled by a  $k$ -subset of  $A$ . The similarity is measured by the sum of squares of the Euclidean distances between the matched points (often called the RMS-distance). A Voronoi-type diagram can be associated with this problem, defining the region of a matching as the set of translations for which the matching minimizes the RMS distance. We show that the complexity of this diagram is  $O(n^{2d}k^d \log^k k)$ . We provide a lower bound of  $\Omega(n^d k^d)$ , for  $k \geq d$ . In the plane and for fixed  $k$ , we prove a tight bound of  $O(n^2)$  leading to an optimal algorithm for the minimization problem.

This is joint work with Matthias Henze and Balázs Keszegh.

**Evanthia Papadopoulou** (Universita della Svizzera Italiana, Lugano)

*On farthest, higher-order, and Hausdorff Voronoi diagrams*

We give an overview of our research in VORONOI/IP7 as related to higher-order and farthest Voronoi diagrams of line segments in two and higher dimensions, as well as the Hausdorff Voronoi diagram, a min-max type of Voronoi diagram.

Farthest and higher-order Voronoi diagrams of line segments illustrate properties surprisingly different from their counterparts of points. For example, a single order- $k$  Voronoi region,  $k > 1$ , may consist of  $\Omega(n)$  disjoint faces. In this talk, I will first summarize structural properties of the order- $k$  Voronoi diagram of line segments in the plane (including disjoint, intersecting, and line segments forming a planar straight line graph). I will then introduce the Gaussian map of a Voronoi diagram as a structure that encodes the signature of the diagram at infinity, and use it as a tool to derive structural properties of farthest line-segment Voronoi diagrams in two, three, and higher dimensions. Using the Gaussian map we establish that the number of Voronoi cells ( $d$ -faces) of the farthest line-segment (and line) Voronoi diagram in  $d$ -dimensions is  $\Theta(n^{d-1})$ .

Work on the farthest Voronoi diagram of line segments in three and higher dimensions is joint with Gill Barequet; in two dimensions with

Sandeep K. Dey. Work on the higher order Voronoi diagram of line segments is joint with Maksym Zavershynskiy. Work on the Hausdorff Voronoi diagram is joint with Panos Cheilaris, Elena Khramtcova, and Stefan Langerman.

**Elena Khramtcova** (Universita della Svizzera Italiana, Lugano)  
*Randomized Incremental Constructions for the Hausdorff Voronoi diagram of point-clusters*

In the Hausdorff Voronoi diagram (HVD) of a set of point-clusters in the plane, the distance between a point  $t$  and a cluster  $P$  is measured as the maximum distance between  $t$  and any point in  $P$ , and the diagram reveals the nearest cluster to  $t$ . If the convex hulls of all clusters are pairwise *non-crossing*, the structural complexity of the diagram is linear. Nevertheless, construction algorithms are far from optimal. In this talk, I will present three randomized incremental construction (RIC) algorithms for the HVD of  $k$  non-crossing clusters of total  $n$  points on their convex hulls, which considerably improve previous results. The first two algorithms are based on maintaining a conflict graph and a history graph, respectively, and run in expected  $O(n \log n \log k)$  time. The third algorithm is based on point-location on a hierarchical data structure, the *Voronoi hierarchy*. The algorithm runs in expected  $O(n \log n + k \log n \log k)$  time. All algorithms require  $O(n)$  expected space. Our algorithms efficiently handle non-standard characteristics of generalized Voronoi diagrams, such as sites of non-constant complexity, sites that are not enclosed in their Voronoi regions, and empty Voronoi regions.

The Hausdorff Voronoi diagram finds direct application in VLSI Critical Area Analysis for computing the probability of fault in a VLSI layout due to random manufacturing defects.

Joint work with Panagiotis Cheilaris, Stefan Langerman, and Evanthia Papadopoulou.

**Maksym Zavershynskiy** (Universita della Svizzera Italiana, Lugano)  
*Randomized algorithms for higher-order Voronoi diagrams*

One of many important generalizations of ordinary Voronoi diagrams is the higher-order Voronoi diagram. The order- $k$  Voronoi diagram is the partitioning of the plane into regions, such that each point within a fixed region has the same  $k$  nearest sites. Many algorithms have been developed that construct the higher-order Voronoi diagram of point-

sites. In this talk we will discuss randomized algorithms that can be used for a larger class of sites—specifically, polygonal objects and the abstract setting. We describe the algorithms in combinatorial rather than geometric terms, which makes it possible to construct higher-order Voronoi diagrams that have bisectors satisfying certain combinatorial properties.

We give a randomized divide-and-conquer algorithm to compute the order- $k$  Voronoi diagram in  $O(kn^{1+\varepsilon})$  basic operations. For solving small subinstances in the divide-and-conquer process, we moreover derive two sub-algorithms to compute the order- $k$  Voronoi diagram in  $O(k^2n \log n)$  and  $O(n^2 2^{\alpha(n)} \log n)$  expected operations. Since each basic operation takes  $O(1)$  time in many situations, such as points in convex distance metrics or the Karlsruhe metric, disjoint line segments or disjoint convex polygons of constant size in the Euclidean metric or under the Hausdorff metric, the three algorithms achieve  $O(kn^{1+\varepsilon})$ ,  $O(k^2n \log n)$ , and  $O(n^2 2^{\alpha(n)} \log n)$  time, respectively, for these concrete order- $k$  Voronoi diagrams.

This is joint work with Cecilia Bohler, Chih-Hung Liu, and Evanthia Papadopoulou.

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**Wednesday, 09:30 - 11:00** (Chair: Ignaz Rutter)

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**Pedro Ramos** (Universidad de Alcalá)

*Shellable drawings and the crossing number of the complete graph*

The Harary-Hill Conjecture states that the number of crossings in any drawing of the complete graph  $K_n$  in the plane is at least  $Z(n) := \frac{1}{4} \lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor \lfloor \frac{n-2}{2} \rfloor \lfloor \frac{n-3}{2} \rfloor$ . In this work, we settle the Harary-Hill conjecture for *shellable drawings*. We say that a drawing  $D$  of  $K_n$  is *t-shellable* if there exist a subset  $S = \{v_1, v_2, \dots, v_t\}$  of the vertices and a region  $R$  of  $D$  with the following property: For all  $1 \leq i < j \leq t$ , if  $D_{ij}$  is the drawing obtained from  $D$  by removing  $v_1, v_2, \dots, v_{i-1}, v_{j+1}, \dots, v_t$ , then  $v_i$  and  $v_j$  are on the boundary of the region of  $D_{ij}$  that contains  $R$ . For  $t \geq n/2$ , we prove that the number of crossings of any  $t$ -shellable drawing of  $K_n$  is at least the long-conjectured value  $Z(n)$ . Furthermore, we prove that all cylindrical,  $x$ -bounded, monotone, and 2-page drawings of  $K_n$  are  $t$ -shellable for some  $t \geq n/2$  and thus they all have at least  $Z(n)$  crossings. The techniques developed provide a unified proof of the Harary-Hill conjecture for these classes of drawings.

This is joint work with Bernardo M. Ábrego, Oswin Aichholzer, Silvia Fernández-Merchant, and Gelasio Salazar.

**Alfredo Garcia** (University of Zaragoza)

*Geometric Biplane Graphs*

This talk is about biplane graphs drawn on a finite point set  $S$  in the plane in general position. This is the family of geometric graphs whose vertex set is  $S$  and can be decomposed into two plane graphs. We show that two maximal biplane graphs—in the sense that no edge can be added while remaining biplane—may differ in the number of edges, and we provide an efficient algorithm for adding edges to a biplane graph to make it maximal. We also study extremal properties of maximal biplane graphs such as the maximum number of edges and the maximum connectivity over  $n$ -element point sets. In particular, we prove that there exist infinitely many point sets  $S$  admitting 11-connected biplane graphs, and no biplane graph is 12-connected. We also show that every sufficiently large set  $S$  admits a 5-connected biplane graph; and there are arbitrarily large point sets that do not admit any 6-connected biplane graph. Finally, we study some connectivity augmentation problems on biplane graphs.

**Birgit Strodthoff** (Johannes Kepler University Linz)

*Boundary-determined Layered Reeb graphs*

Reeb graphs are topological graphs originating in Morse theory, which represent the topological structure of a manifold by contracting the level set components of a scalar-valued function defined on it. The use of more than one function leads to Reeb spaces, which are thus able to capture more features of an object. We introduce the layered Reeb graph as a discrete representation for Reeb spaces of 3-manifolds (possibly with boundaries) with respect to two scalar-valued functions. After that we present a restricted class of defining functions for which the layered Reeb graph can be computed using only a boundary representation of the underlying manifold. This leads to substantial computational advantages if the manifold is given in a boundary description, since no volumetric representation has to be constructed. Finally, we sketch an efficient computation algorithm with some results.

This is joint work with Bert Jüttler.

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**Wednesday, 14:00 - 15:30** (Chair: David Orden)

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**Klavdija Kutnar** (University of Primorska, Koper)

*Half-arc-transitive group actions with a small number of alternets*

A graph  $X$  is said to be  $G$ -half-arc-transitive if  $G \leq \text{Aut}(X)$  acts transitively on the set of vertices of  $X$  and on the set of edges of  $X$  but does not act transitively on the set of arcs of  $X$ . Such graphs can be studied via corresponding alternets, that is, equivalence classes of the so-called reachability relation, first introduced by Cameron, Praeger and Wormald in [*Combinatorica* **13** (1993), 377–396]. If the vertex sets of two adjacent alternets either coincide or have half of their vertices in common the graph is said to be *tightly attached*.

In this talk I will present recent results about graphs admitting a half-arc-transitive group action with at most five alternets: If the number of alternets is at most three, then the graph is necessarily tightly attached, but there exist graphs with four and graphs with five alternets which are not tightly attached. The exceptional graphs all admit a partition giving the rose window graph  $R_6(5, 4)$  on 12 vertices as a quotient graph in case of four alternets, and a particular graph on 20 vertices in the case of five alternets.

This is joint work with Ademir Hujdurović and Dragan Marušič.

**Martin Škoviera** (Comenius University Bratislava)

*Hamilton cycles in truncated triangulations and the maximum genus of a graph*

A truncated triangulation is a 3-valent map on a closed surface  $S$  arising from a triangulation of  $S$  by truncating each vertex, that is, by expanding it into a contractible cycle. Several years ago, Glover and Marušič implicitly employed truncation of highly symmetrical triangulations of orientable surfaces as a tool for proving the existence of Hamilton cycles in large classes of cubic Cayley graphs. We generalise their method by replacing high symmetry with a somewhat surprising weaker condition – upper embeddability of the dual cubic graph. Our result enables us to construct Hamilton cycles in much wider classes of cubic graphs. For example, we show that the truncation of a triangulation of any closed surface with no separating triangle has a Hamilton cycle whenever the number of faces is  $2 \pmod{4}$  and has a cycle missing only two adjacent vertices whenever the number of faces is  $0 \pmod{4}$ .

This is a joint work with Michal Kotrběk and Roman Nedela.

**Sandi Klavžar** (Univerza v Mariboru)

*Two theorems on distances in graphs isometrically embeddable into Cartesian product graphs*

The transitive closure  $\Theta^*$  of the Djoković-Winkler's relation  $\Theta$  turned out to be an extremely useful tool in metric graph theory, cf. [1]. In this talk we will present two recent results (from [2] and [3], respectively) that further apply the relation  $\Theta^*$ .

A weighted graph  $(G, w)$  is a graph  $G = (V(G), E(G))$  together with the weight function  $w : V(G) \rightarrow \mathbb{R}^+$ . The Wiener index  $W(G, w)$  of  $(G, w)$  is:

$$W(G, w) = \frac{1}{2} \sum_{u \in V(G)} \sum_{v \in V(G)} w(u) w(v) d_G(u, v).$$

Our first theorem asserts that  $W(G, w)$  can be expressed as the sum of the Wiener indices of weighted quotient graphs with respect to an arbitrary combination of  $\Theta^*$ -classes.

The  $k$ -th distance moment of a graph  $G$  is defined as

$$W_k(G) = \sum_{\{u, v\} \subseteq V(G)} d(u, v)^k.$$

(Note that  $W_1(G)$  is just the Wiener index of  $G$ ). Our second theorem asserts that the so-called cut-method can be used to compute arbitrary distance moments of all the graphs for which  $\Theta^* = \Theta$  holds (equivalently, for all the graphs that are isometrically embeddable into the Cartesian product of triangles).

- [1] R. L. Graham, P. M. Winkler, On isometric embeddings of graphs, *Trans. Amer. Math. Soc.* 288 (1985) 527–536.
- [2] S. Klavžar, M. J. Nadjafi-Arani, Wiener index in weighted graphs via unification of  $\Theta^*$ -classes, *European J. Combin.* 36 (2014) 71–76.
- [3] S. Klavžar, M. J. Nadjafi-Arani, Computing distance moments on graphs with transitive Djoković-Winkler relation, to appear in *Discrete Appl. Math.*, <http://dx.doi.org/10.1016/j.dam.2013.10.006>.

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**Thursday, 09:30 - 11:00** (Chair: Michael Hoffmann)

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**Chih-Hung Liu** (University of Bonn)

*New results on geodesic and abstract Voronoi diagrams*

Given a set  $S$  of sites in the plane, the  $k^{\text{th}}$ -order Voronoi diagram  $V_k(S)$  is a planar subdivision such that all points in a region share the same  $k$  nearest sites of  $S$ . The number of faces of  $V_k(S)$  is  $O(k(n - k))$  in the Euclidean metric, and the ordinary Voronoi diagram  $V(S)$  is the first-order Voronoi diagram. We present our four new results for Voronoi diagrams. For the  $k^{\text{th}}$ -order geodesic Voronoi diagram in a polygonal domain, we prove that its complexity is  $O(k(n - k) + kc)$ , and the number of its faces is  $O(k(n - k) + kh)$ , where  $c$  is the number of polygonal vertices, and the  $h$  is the number of holes. For the  $k^{\text{th}}$ -order abstract Voronoi diagram, we derive a sharper bound  $2k(n - k)$  for the number of its faces, which applies to many concrete cases. For the abstract Voronoi diagram with disconnected regions, we develop a randomized incremental algorithm with time complexity  $O(s^2 n \sum_{j=2}^n \frac{m_j}{j})$ , where  $s$  is the maximum number of faces for a Voronoi region and  $m_j$  is the average number of faces per region over all abstract Voronoi diagrams of  $j$  sites. For the abstract Voronoi diagram in a domain whose structure is a forest and whose bisecting system satisfies certain constraints, we propose a linear-time algorithm.

**Gabriela Majewska** (University of Warsaw)

*Generalized beta-skeletons*

The  $\beta$ -skeletons  $\{G_\beta(V)\}$  for a point set  $V$  is a hierarchy of graphs on  $V$  based upon a natural notion of “neighborliness” parametrized by a real number  $\beta$ . They are both important and popular because of many practical applications which span a spectrum of areas from geographic information systems to wireless ad hoc networks and machine learning. Two types of  $\beta$ -skeletons are especially well-known, the Gabriel Graph ( $GG$ ), for  $\beta = 1$ , and the Relative Neighborhood Graph ( $RNG$ ), for  $\beta = 2$ .

In our presentation we show a more general definition of  $\beta$ -skeletons only based on a distance criterion. This criterion allows us to define  $\beta$ -skeletons for a bigger class of events, for example in weighted graphs and for a set of objects. We show that in each of those cases there are established relations between  $\beta$ -skeletons and other graphs (e.g.,

minimum spanning tree) which also occur for the previous definition for a set of points.

**Gernot Walzl** (Graz University of Technology)

*Straight Skeletons in 3-space*

The straight skeleton of a polyhedron is defined by a shrinking (off-setting) process where each facet is parallelly shifted inwards. At the very first moment, vertices of degree  $> 3$  are decomposed into vertices of degree  $= 3$ . During the shrinking process, the edges of the polyhedron move on the angular bisector planes of their incident facets and trace out the facets of the straight skeleton. The polyhedron undergoes changes of geometric, combinatorial and topological nature.

This talk focuses on ambiguity, existence of a solution and construction algorithms. We provide examples and animations to visualize our results.

This is joint work with Franz Aurenhammer.

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**Thursday, 14:00 - 15:30** (Chair: Martin Škoviera)

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**Jurij Kovič** (University of Primorska, Koper)

*Petrie maps and Petrie configurations*

We present a method for obtaining examples of  $(v_3)$  configurations from flag graphs of maps.

*Combinatorial  $(v_k)$  configurations* are incidence structures consisting of  $v$  points and  $v$  lines, such that each of the lines is incident with  $k$  points and each of the points is incident with  $k$  lines. They are more general concept than *geometric  $(v_k)$  configurations* whose lines and points correspond to lines and points in the Euclidean plane.

*Maps* are cellular embeddings of graphs into compact surfaces.

*Flag graphs*  $F(\mathcal{M})$  are combinatorial descriptions of maps  $\mathcal{M}$ . They are 3-valent graphs whose edges are labeled 0,1,2, satisfying the following property: if the 0-adjacent, 1-adjacent and 2-adjacent vertices (= flags) of a flag  $\Phi$  are denoted  $\Phi^0, \Phi^1, \Phi^2$ , then  $(\Phi^0)^2 = (\Phi^2)^0$ .

We show how for any permutation  $(i, j, k)$  of  $(0, 1, 2)$  the *Petrie walks*  $P_{i,j,k}(\Phi) = (\Phi, \Phi^i, (\Phi^i)^j, ((\Phi^i)^j)^k, (((\Phi^i)^j)^k)^i \dots$  in the flag graphs  $F(\mathcal{M})$  of maps  $\mathcal{M}$  may be used for obtaining examples of  $(v_3)$  configurations. The maps that allow such a construction are called *Petrie maps*, and the corresponding configurations are called *Petrie configurations*. For example, the maps of all Platonic solids are Petrie maps.

We give an example of a  $(24_3)$  configuration obtained from a tetrahedron and present some structural properties of Petrie configurations.

**Tomaz Pisanski** (University of Ljubljana and of Primorska, Koper)  
*A plausible model for self-assembly of polyhedral shapes from linear chains*

Recently a group of Slovenian scientists and mathematicians under the leadership of Roman Jerala produced a polypeptide string that can self-assemble in the shape of a stable tetrahedron in such a way that each tetrahedral edge is composed of two intertwined peptide segments. We will give a mathematical interpretation of this remarkable bioengineering task. The model that best describes a self-assembly polyhedron comes from topological graph theory. It can be interpreted, on the one hand, as a gluing process turning a fundamental polygon into a closed surface and on the other hand, as an Eulerian trail in a doubled skeleton graph of the corresponding polyhedron. The design of such polypeptides requires a solution of several interesting combinatorial problems and problems of combinatorial optimization. The talk is based on work in progress with Nino Bašič and Roman Jerala.

**Jelena Sedlar** (University of Split)  
*Remoteness, proximity and few other distance invariants in graphs*

We establish maximal trees and graphs for the difference of average distance and proximity proving thus the corresponding conjecture posed in [1]. We also establish maximal trees for the difference of average eccentricity and remoteness and minimal trees for the difference of remoteness and radius proving thus that the corresponding conjectures posed in [1] hold for trees.

- [1] M. Aouchiche, P. Hansen. Proximity and remoteness in graphs: results and conjectures. *Networks*, 58(2):95–102, 2011.

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**Thursday, 16:00 - 18:00** (Chair: Rolf Klein)

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**Bartosz Walczak** (Jagiellonian University Krakow)  
*Coloring geometric intersection graphs via on-line games*

The on-line graph coloring problem is modeled by a game between two players: Presenter, who constructs a graph of some fixed class present-

ing new vertices one by one, and Algorithm, who colors the vertices right after they are presented without the possibility of changing the color afterwards. The goal of Algorithm is to use as few colors as possible, while Presenter tries to force Algorithm to use many colors. Any game of this kind gives rise to a new class of graphs, so-called game graphs, which “encode” strategies of Presenter in the game. The number of colors that Algorithm is forced to use playing against such a strategy is equal to the chromatic number of the corresponding game graph. It turns out that game graphs of appropriately defined games can be characterized as intersection graphs of geometric objects. This allows us to provide lower and upper bounds on the maximum possible chromatic number of geometric intersection graphs by analyzing the corresponding on-line coloring games. I will present a survey of results obtained with this method and a detailed example of its application for constructing triangle-free geometric intersection graphs with large chromatic number.

**Jarek Grytczuk** (Jagiellonian University Krakow)

*Graph coloring with geometric flavor*

There exist lots of intriguing coloring problems involving graphs and geometry. Typically, a class of graphs is defined on some geometric structure and then various coloring parameters for this class are studied. A representative issue of this type is the famous Hadwiger-Nelson problem concerning the chromatic number of the plane. In recent years many new types of graph colorings arose (game coloring, nonrepetitive coloring, clique coloring, etc.), and it is also interesting to investigate them from a geometric point of view. I will present some of my favorite problems and results in this area.

**Alexander Wolff** (Universität Würzburg)

*Angular Schematization – Some GraDR Results*

Angular schematization (GraDR work package WP02) deals with (constructing and) drawing networks under angular restrictions. Consider, for example, metro maps that are usually drawn such that vertices roughly keep their relative positions and edges are laid out in an *octilinear* fashion, that is, they are drawn as horizontal, vertical, or  $45^\circ$ -diagonal straight-line segments.

Such angular restrictions have led to a variety of interesting problems in graph drawing, information visualization, geographic information sci-

ence, computational geometry, VLSI layout or underground mining. In some of these communities (such as graph drawing or VLSI layout), rectilinear (“orthogonal”) connections have a long history, but recently octilinear or curvilinear connections have moved into the spotlight, bringing with them completely new problems and challenges. In other fields of application such as underground mining, the set of allowed slopes is not discrete, but there is an upper bound on the maximum slope.

I will report on some of the following results that have been achieved within the GraDR project. For example, we have come up with new approaches to draw metro maps using Bézier curves [GD’12] or circular arcs, possibly concentric [Schematic Mapping Workshop’14]. For a network design problem, the (*generalized*) *minimum Manhattan network problem*, we have devised approximation algorithms that compute light networks for connecting points in the plane, but also in higher dimensions [ESA’11, ISAAC’13]. We have investigated the complexity of schematizing embedded paths under order constraints [CGTA’14]. We have explored ways to schematize subdivisions (think of political maps) preserving the areas of the countries [GiScience’10, ACMGIS’11, ACMGIS’13]. We have devised new algorithms for *boundary labeling* [InfoVis’12, WADS’13, GD’13], where the aim is to connect a given set of points within an axis-parallel rectangle (the map) using rectilinear edges (“leaders”) to labels that lie outside the map, but touch its boundary. We have considered *monotone* straight-line drawings [GD’11, Algorithmica’13], where we require for each pair of vertices that there exists a direction  $d$  such that the drawing contains a path that connects the vertices and is increasing in  $d$ . We have shown how to make curved schematization topologically safe [Cartogr. J.’13, PacificVis’14]. Last but not least, we have introduced two new variants of the well-established orthogonal drawing style, namely *smooth drawings* [GD’12, LATIN’14] and *slanted drawings* [GD’13], that potentially lower the visual complexity of the layout by reducing the number of bends or inflection points.

This is joint work with colleagues from IP2 (Tübingen), AP1-IT (Roma), AP2-NL (Eindhoven), AP3-DE (Karlsruhe), and AP4-DE (Münster).

**Mario Kapl** (Johannes Kepler University Linz)

*Medial Axis Regularization via Total Curvature Variation Fairing*

We present a new fairing method for planar curves which is particularly well suited for the regularization of the medial axis of a planar

domain. The fairing algorithm is based on the concept of total variation regularization which has originated in image processing. The original boundary, which is given as a closed B-spline curve, is approximated by another curve that possesses a smaller number of curvature extrema. Consequently, the modified curve leads to a smaller number of branches of the medial axis.

In order to compute the medial axis, we use the state-of-the-art algorithm from [1] which is based on arc spline approximation and a domain decomposition approach. We improve this algorithm by using a different decomposition strategy that allows to reduce the number of base cases from 13 to only 5, and reduces the number of conic arcs in the output. The performance of total curvature variation fairing for medial axis regularization is demonstrated by several examples and compared with an existing fairing method.

This is joint work with Florian Buchegger and Bert Jüttler.

- [1] O. Aichholzer, W. Aigner, F. Aurenhammer, T. Hackl, B. Jüttler, and M. Rabl. Medial axis computation for planar free-form shapes. *Computer-Aided Design*, 41:339–349, 2009.

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**Friday, 09:30 - 11:00** (Chair: Alexander Wolff)

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**Andrei Asinowski** (Freie Universität Berlin)

*Disjoint compatibility of non-crossing matchings for points in convex position*

Let  $X_{2k}$  be a set of  $2k$  labeled points in convex position in the plane. We consider non-crossing straight-line perfect matchings of  $X_{2k}$  (the number of such matchings is the  $k$ th Catalan number). Two such matchings,  $M$  and  $M'$ , are *disjoint compatible* if they do not have common edges, and no edge of  $M$  crosses an edge of  $M'$ . Denote by  $\text{DCM}_k$  the reconfiguration graph of non-crossing matchings of  $X_{2k}$  with respect to disjoint compatibility – that is, the graph whose vertices correspond to such matchings, and two vertices are adjacent if and only if the corresponding matchings are disjoint compatible. We study such graphs from the structural point of view, focusing on the structure, the size and the number of their connected components. We show that for each  $k \geq 9$ , the connected components of  $\text{DCM}_k$  form exactly three isomorphism classes – namely, there are several isomorphic “small components”, several isomorphic “middle-size components”, and one “big

component”. Moreover, we determine precisely the number and the structure of small and medium components (they follow different patterns for odd and for even values of  $k$ ), and characterize the matchings that belong to these components.

This is joint work with Oswin Aichholzer and Tillman Miltzow.

**Aaron Dall** (Universitat Politècnica de Catalunya, Barcelona)

*A Polyhedral Proof of the Matrix Tree Theorem*

The Matrix-Tree Theorem is a classical result in algebraic graph theory that gives a relation between the number of spanning trees of a connected graph and the product of the nonzero eigenvalues of its Laplacian matrix. We give a new proof of this result by showing that two lattice zonotopes associated to the graph have the same volume.

**Manuel Wettstein** (ETH Zurich)

*Counting and Enumerating Crossing-free Perfect Matchings*

We present a simple counting algorithm that computes the number of crossing-free straight-line perfect matchings on a planar point set of size  $n$  in time  $O(n^3 2^n)$ . We then show how a small adaptation results in an efficient enumeration algorithm, where by efficient we mean polynomial time delay for each enumerated object.

The presented ideas can be applied to various other families of crossing-free structures, such as all crossing-free geometric graphs, crossing-free spanning trees, and crossing-free spanning cycles. If time permits, we quickly outline the corresponding results.

