Randomized Incremental Constructions for the Hausdorff Voronoi diagram of point-clusters

Elena Khramtcova[1]

joint work with Panagiotis Cheillaris[1]
Stefan Langerman[2] and Evanthia Papadopoulou[1]

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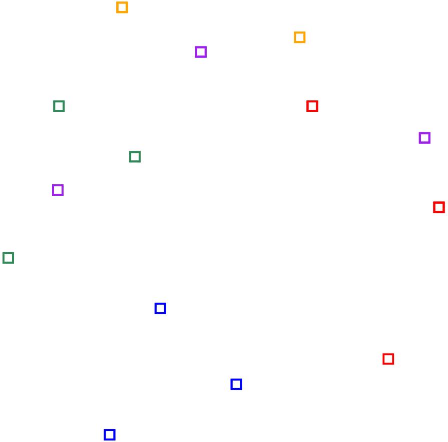
[2]Université Libre de Bruxelles, Belgium

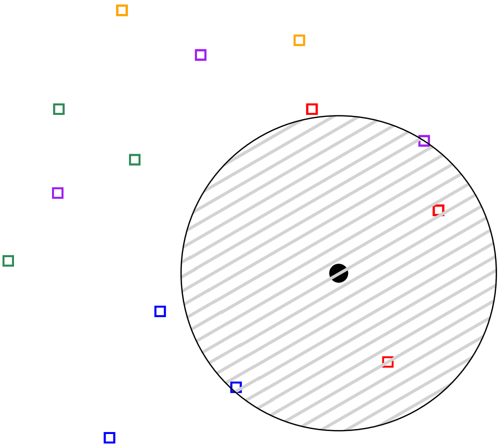
February 18, 2014
Berlin, Germany

Fault-tolerance of a VLSI design



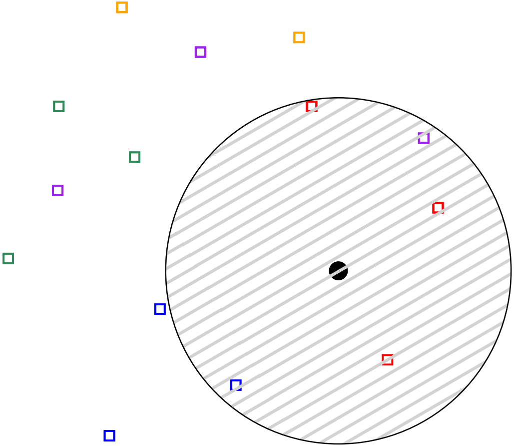
Fault-tolerance of a VLSI design

No harm



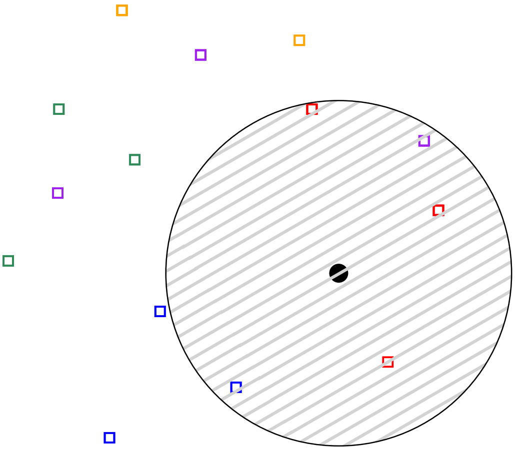
Fault-tolerance of a VLSI design

The red element
is destroyed!



Fault-tolerance of a VLSI design

What is the critical size of a defect in each point?



Hausdorff Voronoi diagram (HVD)

Space: $\mathbb{R}^2$

Sites: point-clusters

Distance: $d_f(q, R) = \max_{r \in R} d(q, r)$

Type: nearest-neighbor diagram (*MIN-MAX* diagram)

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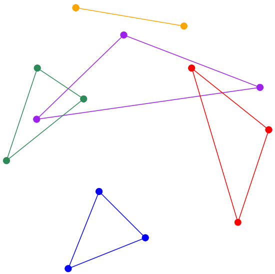

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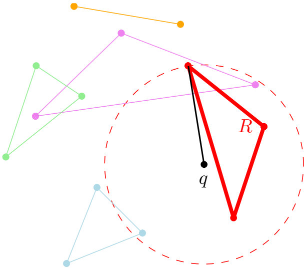

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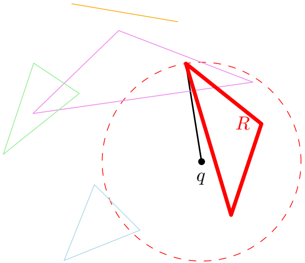

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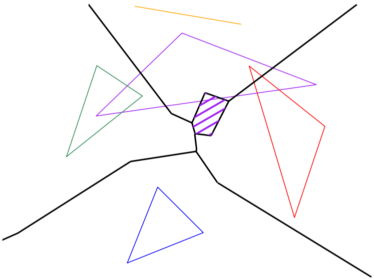

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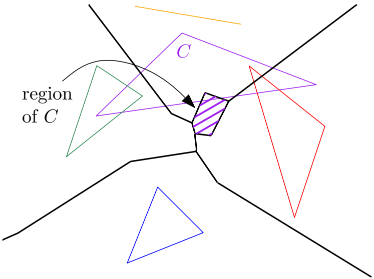

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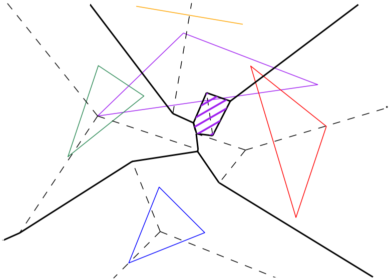

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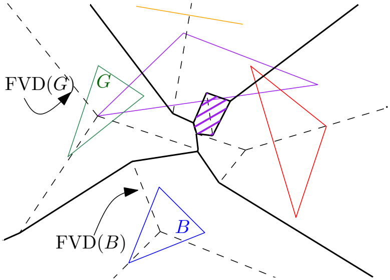

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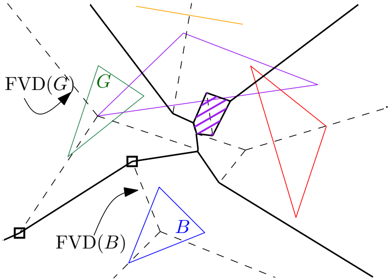

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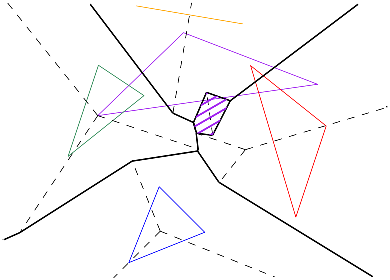
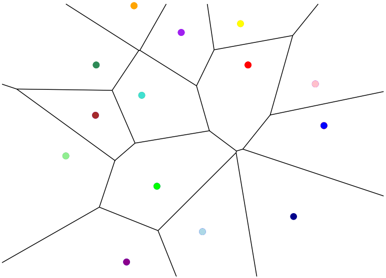


k - number of clusters;
n - total number of points on their convex hulls.

HVD generalizes VD

- If $k = n$: nearest-neighbor Voronoi diagram
- If $k = 1$: farthest Voronoi diagram



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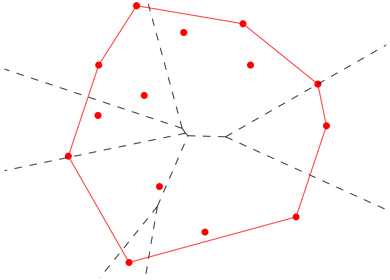

Structural complexity of the HVD

The HVD may have size $\Theta(n^2)$ Papadopoulou'04

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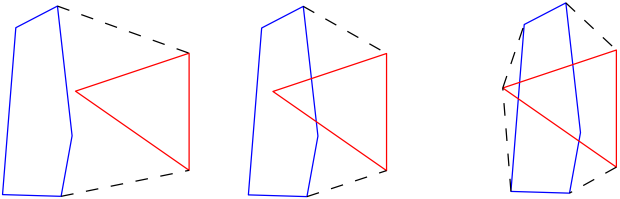
Size depends on *crossings* of clusters

Non-crossing clusters

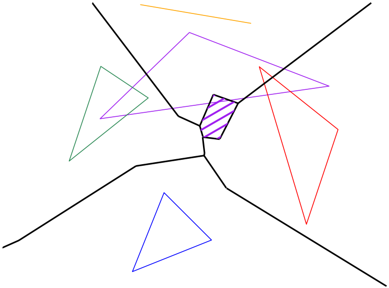


non-crossing crossing

HVD of non-crossing clusters

- Connected regions, $O(n)$ size

Edelsbrunner et al.'89; Papadopoulou'04



Construction of HVD (non-crossing clusters)

Method	Time	Ref.
Lifting transformation	$O(n^2)$	Edelsbrunner et al.'89
Rand. Incremental based on AVD	exp. $O(bn \log n)$	Klein et al.'93; Abellanas et al.'97
Plane sweep	$O((n+K)\log n)$	Papadopoulou'04
Divide & Conquer	$O((n+K)\log n)$	Papadopoulou & Lee'04
Divide & Conquer	$O(n \log^4 n)$ $O(n \log^2 n)$ space	Dehne et al.'06

- b: time to compute a bisector ($O(n)$ for clusters of linear size).
- K: # of *interacting clusters* ($K = O(n^2)$, small in practice).

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- Insert sites in random order;
- Update the diagram after each insertion;
- Maintain a helper data structure.

Conflict graph/History graph — Clarkson & Shor'89

Voronoi hierarchy — Devillers'02; Karavelas & Yvinec'03

CGAL library

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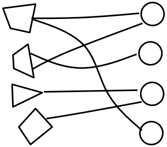
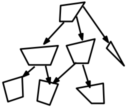


elements of the diagram

sites not yet inserted



leaves are elements of current diagram

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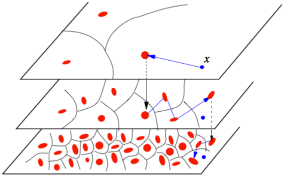

Conflict/History graph: requirements

- The work to update the conflict graph is proportional to number of edges deleted or created.
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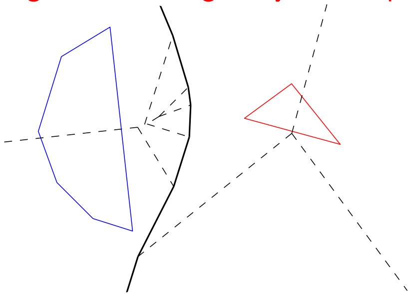

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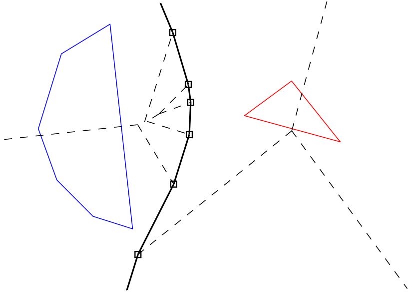
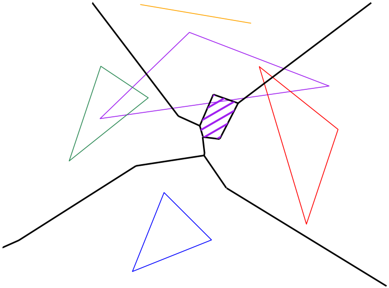


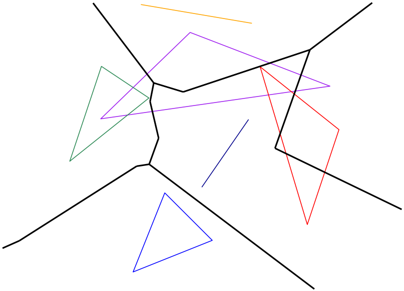
Voronoi hierarchy for HVD: an issue

A site may be not enclosed in its region.



Voronoi hierarchy for HVD: an issue

A region may become empty.



Our results

Method	Expected time	Expected space
Augmenting *Voronoi hierarchy*	$O(n \log n \log k)$	O(n)
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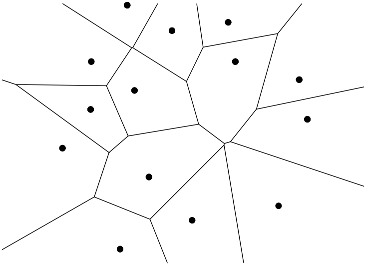
Standard definition of conflict

For the nearest-neighbor VD of points (and for AVDs):



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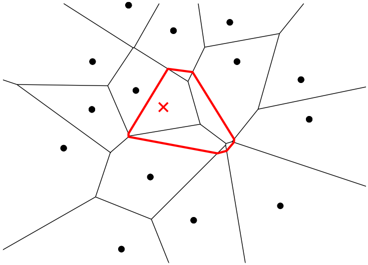

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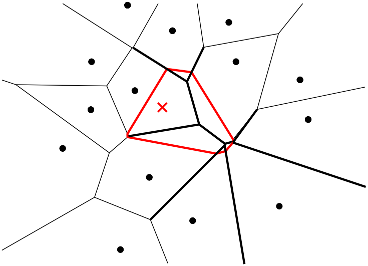

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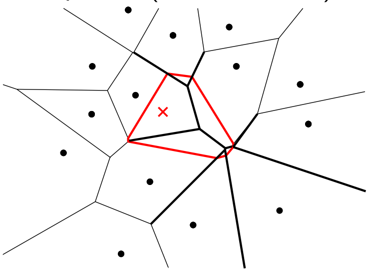

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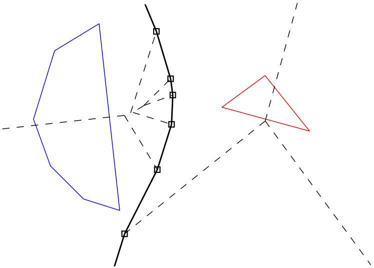
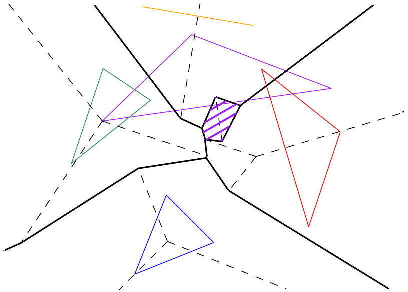


One update may take $\Omega(n)$

Definition of conflict for the HVD

Property: Voronoi region of C (hreg(C)) intersects FVD(C) in a single connected component.

Conflict: Element of the current HVD that contains the *highest point* of hreg(C)∩FVD(C).



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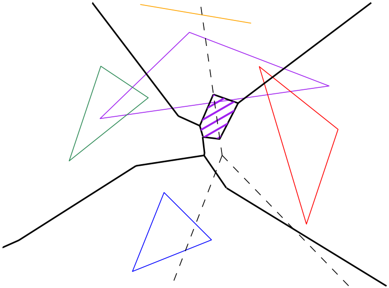

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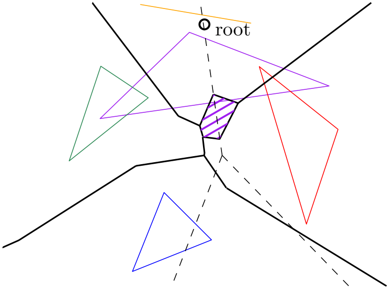

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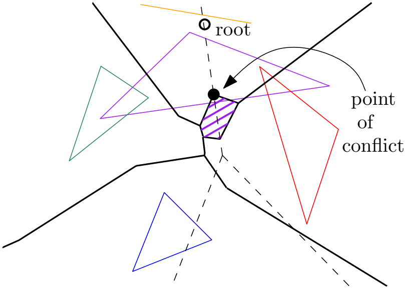
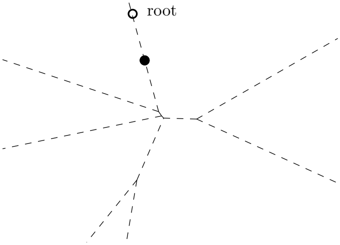


Updating a conflict for the HVD

Relocate the point of conflict



$O(\log n)$ time with *Visibility-based decomposition* of the HVD

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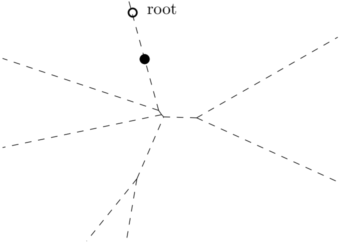

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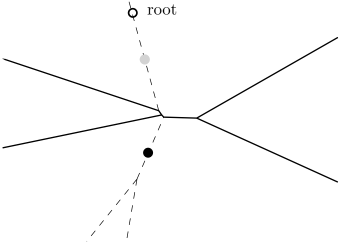
Find the new point of conflict w.r.t. the last inserted cluster



$O(\log n)$ time per edge using *Separator decomposition* of FVD(C).

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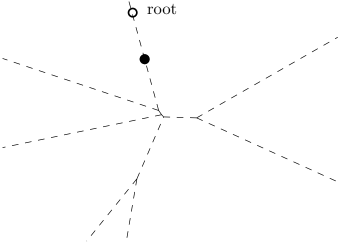

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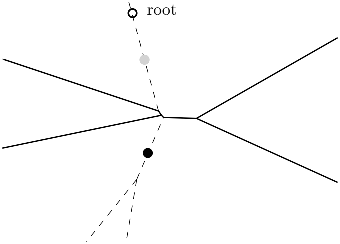

$O(\log n)$ time per edge using *Separator decomposition* of FVD(C).

Time: $O(\log n)(1 + \text{number of pruned edges of FVD}(C))$

Summary and future work

Method	Expected time	Expected space
Augmenting Voronoi hierarchy	$O(n \log n \log k)$	O(n)
Conflict/history graph	$O(n \log n + k \log n \log k)$	$O(n)$

Future work:

- Arbitrary point-clusters
- Clusters of line-segments

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